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Finite time L_1 Approach for Missile Overload Requirement Analysis in Terminal Guidance

Ji Denggao*, He Fenghua, Yao Yu

Control and Simulation Center, Harbin Institute of Technology, Harbin 150080, China

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Abstract

This article analyzes the problem about the missile overload requirement in a homing terminal guidance under various engagement scenarios. An augmented proportional navigation guidance (APNG) model is introduced on the basis of linear kinematics. To analyze the peak-to-peak performance of the terminal guidance system, a new finite time L_1 performance measure for a linear time-varying (LTV) continuous system is proposed. Then, according to the idea of the adjoint system, a novel method for computing the L_1 norm of a linear continuous system is first derived. Within the finite time L_1 framework, the quantitative relation between the guidance loop dynamics and the maximum missile-target maneuver ratio is offered. This relation is expressed in the form of graphs and formulas that can be used to synthesize some of the major subsystem specifications for the missile guidance system. The illustrative examples show that a significant performance improvement is achieved with the proposed guidance loop dynamics.

Keywords: finite time; L_∞-induced norm; performance; time varying systems; missile overload requirement; terminal guidance

1. Introduction

In the missile guidance process, the missile overload in the terminal phase is critical to the resulting engagement outcome. In practice, missile subsystems, such as autopilot and guidance systems, are usually designed separately. After combining them together, analysis of missile overload requirement would become very difficult and is thus a severe challenge to control engineers because of their interactive effects and uncertainties[1-4].

In the missile terminal guidance process, the external input of the guidance system is the target maneuver with limited acceleration amplitude. Moreover, the major source of the missile overload is the target maneuver. In engineering practice, to design nonsaturating guidance systems, the guidance system designers need to know the missile maximum overload requirement against the target maneuver[5]. Therefore, the maximum missile-target maneuver ratio under various engagement scenarios should be analyzed since the maximum acceleration of the target is relatively easier to be estimated than other maneuver information of the target. In Ref.[5], the maximum missile-target maneuver ratio is obtained under some assumptions on the guidance loop dynamics. This study proposes a theoretical and quantitative approach to analyze the missile maximum overload requirement for the worst-case target maneuver without any assumption.

It is also noted that there are some salient features in the terminal guidance analysis quite different from the conventional performance analysis. First, the plant is linear time-varying (LTV) and possibly nonlinear while in most performance analyses[6-8], since the nominal plant is assumed linear time-invariant (LTI), for the nonlinear and time-varying effects are absorbed by the uncertainty block. Second, missile guidance is such a complicated control problem that only the system behaviors in a finite time interval should be considered[9]. It means that these special physical problems need to be solved within a new framework.

First introduced by M. Vidyasagar[10], the L_1 approach formulated the problem of optimal disturbance rejection in the case of the disturbance which generated as an output of a stable system in response to an input which was assumed to be either of unit amplitude or arbitrary. This approach is of considerable practical importance because it aims to minimize the maximum value of the system error. In the classical

*Corresponding author. Tel.: +86-451-86400627.
E-mail address: jidg@163.com
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control system performance analysis, such as L_∞ norm minimization, an underlying assumption is that the system external inputs have their power in finite frequency ranges and are bounded energy signals. From this point of view, the L_1 approach serves as a complement to the theory of L_∞ norm minimization.

This article attempts to analyze missile overload requirement against the worst-case target maneuver based on the guidance loop dynamics over a finite time interval with the novel finite time L_1 approach. First, the L_1 performance measure of a LTV system is generalized by introducing the notion of finite time LTV system behaviors. Then, an effective algorithm of the L_∞-induced norm for a LTV continuous system is first proposed by constructing the adjoint system that associates with the original system at a given moment. Further, this algorithm can also be employed to the continuous system with any of the following characteristics: infinite time, finite time, LTI, single-input-single-output (SISO), and multi-input-multi-output (MIMO). Based on the LTV terminal guidance model, the quantitative relation between the maximum missile-target maneuver ratio and guidance loop dynamics, such as effective navigation ratio, guidance time constant, and the modified coefficient in augmented proportional navigation guidance (APNG), is studied with the novel finite time L_1 approach. In addition, the analysis and simulation results provide the feasible theoretical basis for the design of a missile terminal guidance system.

2. Mathematical Modeling

Fig.1 shows a missile terminal guidance loop which typically includes the dynamics of a seeker, a guidance computer, and missile dynamics[11]. In this work, the seeker is simply viewed as a pure differentiator which generates the line-of-sight (LOS) rate of the target without delay. The guidance computer is viewed as an algebraic function that computes the acceleration command with the information from the seeker. Missile/autopilot dynamics that includes vehicle dynamics, autopilot, and actuator is viewed as a simple first-order or second-order system.

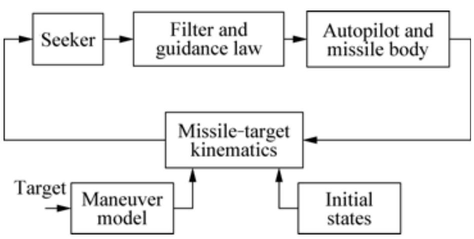

Fig.1 Missile terminal guidance system.

The general formulation of a 3D interception problem is rather complicated. However, on the assumption that the lateral and longitudinal maneuver planes are decoupled by means of roll control, the 3D problem may quite reasonably be treated as an equivalent 2D one. Fig.2 shows the kinematic model for a planar interception when a homing missile intercepts a maneuvering target, where, R represents the relative range between the target and missile along the LOS, q is the LOS angle, and a_m and a_t are the missile and target accelerations normal to the LOS, respectively. The kinematic relation between the target and missile motion can be described by the nonlinear equations:

$$R\ddot{q} + 2\dot{R}\dot{q} = a_t - a_m \tag{1}$$

$$\ddot{R} - R\dot{q}^2 = a_{tr} - a_{mr} \tag{2}$$

where a_{mr} and a_{tr} represent the missile and target accelerations along the LOS, respectively. If the missile approaches the target with a constant closing speed V_c, Eq.(2) can be ignored and the range R is simply expressed by $R=V_c t_{go}$, where t_{go} denotes the time taken by the missile to intercept the target.

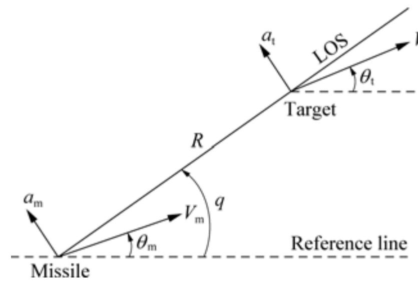

Fig.2 Planar missile and target engagement geometry.

It is well known that the most advanced guidance law, regarded as the augmented proportional navigation guidance law, is compounded of the proportional navigation guidance law and the target acceleration compensation. Then, the guidance command a_{mc} is derived from

$$a_{mc} = NV_c\dot{q} + \frac{K}{2}a_t \tag{3}$$

where N is the effective navigation ratio, often chosen to be 3-5 in practice, and K is the modified coefficient to compensate the target maneuver normal to the LOS.

Missile/autopilot dynamics may be represented by

$$\dot{x}_m = F_m x_m + G_m a_{mc} \tag{4}$$

$$a_m = H_m x_m \tag{5}$$

or

$$a_m = G(s)a_{mc} \tag{6}$$

where $\{F_m, G_m, H_m\}$ is the state-space realization of the missile/autopilot dynamics, and $G(s)$ denotes the transfer function.

For simplicity, the missile/autopilot dynamics is viewed as a first-order system, but it should be pointed out that any linear high order missile/autopilot system is also allowed within the finite time L_1 framework. By combining Eq.(1) with Eqs.(3)-(5), the APNG loop dynamics can be expressed by the following linear

time-varying differential equations:

$$\left.\begin{aligned}&\begin{bmatrix}\ddot{q}\\ \dot{a}_{\mathrm{m}}\end{bmatrix}=\begin{bmatrix}\dfrac{2}{t_{\mathrm{e}}-t} & \dfrac{-1}{V_{\mathrm{c}}\left(t_{\mathrm{e}}-t\right)}\\ \dfrac{NV_{\mathrm{c}}}{T} & \dfrac{-1}{T}\end{bmatrix}\begin{bmatrix}\dot{q}\\ a_{\mathrm{m}}\end{bmatrix}+\begin{bmatrix}\dfrac{1}{V_{\mathrm{c}}\left(t_{\mathrm{e}}-t\right)}\\ \dfrac{K}{2T}\end{bmatrix}a_{\mathrm{t}}\\ &z=\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}\dot{q}\\ a_{\mathrm{m}}\end{bmatrix}\end{aligned}\right\}\tag{7}$$

where T is the time constant of the missile/autopilot, which is equivalent to a state-space realization with $F_{\mathrm{m}}=-1/T$, $G_{\mathrm{m}}=1/T$, and $H_{\mathrm{m}}=1$, and t_{e} is the total flight time of the engagement. Note that t_{e} is now a constant. The value of $(t_{\mathrm{e}}-t)$ is the time to go until the end of the flight.

From Eq.(7), it is noted that the missile/target engagement in the terminal phase can be regarded as an LTV system in the finite time interval $[0, t_{\mathrm{e}})$. In this article, the output of interest is the missile lateral acceleration throughout the flight.

3. Finite time L_1 Performance Analysis of LTV System

This section will present the finite time L_1 performance measure and describe the way it is calculated for a given LTV continuous system.

3.1. Performance measure

In this subsection, the performance measure, the L_∞-induced norm, is defined for an LTV continuous system with a special emphasis on finite horizon behaviors. Various definitions for LTV systems presented in this article are borrowed from Ref.[12].

Eq.(7) can easily be converted into the canonical LTV form as follows:

$$\Sigma:\begin{cases}\dot{\boldsymbol{x}}(t)=\boldsymbol{A}(t)\boldsymbol{x}(t)+\boldsymbol{B}(t)\boldsymbol{w}(t),\boldsymbol{x}(0)=\boldsymbol{0}\\ z(t)=\boldsymbol{C}(t)\boldsymbol{x}(t)\end{cases}\tag{8}$$

where $\boldsymbol{x}(t)\in\mathbf{R}^n$ is the system state vector, $\boldsymbol{w}(t)\in L_2[0,t_{\mathrm{f}}]$ the exogenous disturbance signal, $\boldsymbol{x}(0)$ the initial state, $z(t)\in\mathbf{R}^m\subset\mathbf{R}^n$ the state combination (objective function signal) to be attenuated, and $\boldsymbol{A}(t)$, $\boldsymbol{B}(t)$, $\boldsymbol{C}(t)$ the bounded continuous matrices.

To formalize the definition of the finite time L_1 performance measure, Definition 1 establishes the notation for L_∞ signal norms over the finite horizon used throughout the article.

Definition 1 For a finite constant $t_{\mathrm{f}}>0$, the finite-horizon infinity norm of a signal $f:\mathbf{R}^+\rightarrow\mathbf{R}^+$ is defined as

$$\|f\|_{\infty,[0,t_{\mathrm{f}}]}:=\sup_{t\in[0,t_{\mathrm{f}}]}\left\|f\left(t\right)\right\|\tag{9}$$

where $\|\bullet\|$ denotes the Euclidean L_2 vector norm. The linear space $L_\infty[0, t_{\mathrm{f}}]$ is expressed by

$$L_\infty[0,t_{\mathrm{f}}]:=\left\{f:\mathbf{R}^+\rightarrow\mathbf{R}^+\left|\|f\|_{\infty,[0,t_{\mathrm{f}}]}<\infty\right.\right\}\tag{10}$$

The subset $\left\{f:\mathbf{R}^+\rightarrow\mathbf{R}^+\left|\|f\|_{\infty,[0,t_{\mathrm{f}}]}\leq 1\right.\right\}\subset L_\infty\left[0,t_{\mathrm{f}}\right]$ is denoted by $BL_\infty[0,t_{\mathrm{f}}]$.

For a fixed time $t_{\mathrm{f}}>0$, the closed-loop L_1 performance measure of the system Eq.(8) is defined as

$$J\left(\Sigma,t_{\mathrm{f}}\right)=\sup_{\boldsymbol{w}(t)\in BL_\infty[0,t_{\mathrm{f}}]}\|z(t)\|_{\infty,[0,t_{\mathrm{f}}]}\tag{11}$$

Here, under the fixed initial condition, $\boldsymbol{x}(0)=\boldsymbol{0}$, Eq.(11) defines a mapping from bounded amplitude inputs $\boldsymbol{w}(t)\in BL_\infty[0, t_{\mathrm{f}}]$ to bounded amplitude penalty outputs $z(t)\in BL_\infty[0, t_{\mathrm{f}}]$ and a relevant performance criterion is the peak-to-peak or L_∞-induced norm of this mapping.

In the LTI case, Eq.(11) can be derived from

$$J(\Sigma,t_{\mathrm{f}})=\int_0^{t_{\mathrm{f}}}|h(t)|\mathrm{d}t\tag{12}$$

where $h(t)$ is the known unit impulse response of the LTI system[13]. However, in the LTV case, Eq.(12) becomes complicated as follows:

$$J(\Sigma,t_{\mathrm{f}})=\sup_{t\in[0,t_{\mathrm{f}}]}\int_0^t|h(t,\tau)|\mathrm{d}\tau\tag{13}$$

where $h(t,\tau)$ is the impulse response of the LTV system. Since $h(t,\tau)$ has no general analytic expression, Eq.(13) can not be used to compute the L_∞-induced norm for LTV system in practice. To the best of authors' knowledge, there is no mention in the literature of the computation method of the L_∞-induced norm for a LTV system at present.

3.2. Computation method of L_∞-induced norm for LTV system

Let t_{o} $(0\leq t_{\mathrm{o}}\leq t_{\mathrm{f}})$ be the moment when the output of the system is of particular interest. The adjoint response of system Eq.(8) at t_{o} is defined as

$$\left.\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t_{\mathrm{g}}}\boldsymbol{x}^{\mathrm{adj}}(t_{\mathrm{g}})=\boldsymbol{A}^{\mathrm{T}}(t_{\mathrm{o}}-t_{\mathrm{g}})\boldsymbol{x}^{\mathrm{adj}}(t_{\mathrm{g}})\\ &\boldsymbol{z}^{\mathrm{adj}}(t_{\mathrm{g}})=\boldsymbol{B}^{\mathrm{T}}(t_{\mathrm{o}}-t_{\mathrm{g}})\boldsymbol{x}^{\mathrm{adj}}(t_{\mathrm{g}})\\ &\boldsymbol{x}^{\mathrm{adj}}(0)=\boldsymbol{C}^{\mathrm{T}}(t_{\mathrm{o}})\end{aligned}\right\}\tag{14}$$

where $\boldsymbol{A}^{\mathrm{T}}$, $\boldsymbol{B}^{\mathrm{T}}$ and $\boldsymbol{C}^{\mathrm{T}}$ denote the transpose of matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$, respectively; $\boldsymbol{x}^{\mathrm{adj}}$ is the state adjoint response; $\boldsymbol{z}^{\mathrm{adj}}$ the output adjoint response, t_{g} the time variable of the system Eq.(14). System Eq.(14) can also be regarded as the auxiliary system used to compute the L_∞-induced norm of the linear continuous system.

With the aid of system Eq.(14), the following theorem can be obtained.

Theorem 1 The output of system Eq.(8) at the given moment t_{o} is

$$z(t_{\mathrm{o}})=\int_0^{t_{\mathrm{o}}}\left(\boldsymbol{z}^{\mathrm{adj}}(t_{\mathrm{o}}-\tau)\right)^{\mathrm{T}}\boldsymbol{w}(\tau)\mathrm{d}\tau\tag{15}$$

Proof The output of system Eq.(8) at the given moment t_{o} can be written into

$$z(t_o)=\int_0^{t_o}\boldsymbol{C}(t_o)\boldsymbol{\Phi}(t_o,\tau)\boldsymbol{B}(\tau)\boldsymbol{w}(\tau)\mathrm{d}\tau \tag{16}$$

where $\boldsymbol{\Phi}(t_o,\tau)$ is the transition matrix of system Eq.(8). It is noted that the transition matrix by its definition satisfies the following equations:

$$\frac{\partial}{\partial t}\boldsymbol{\Phi}(t,\tau)=\boldsymbol{A}(t)\boldsymbol{\Phi}(t,\tau) \tag{17}$$

$$\frac{\partial}{\partial \tau}\boldsymbol{\Phi}(t,\tau)=-\boldsymbol{\Phi}(t,\tau)\boldsymbol{A}(\tau) \tag{18}$$

$$\boldsymbol{\Phi}(t,t)=\boldsymbol{I} \tag{19}$$

$$\boldsymbol{\Phi}(t,\tau)\boldsymbol{\Phi}(\tau,\xi)=\boldsymbol{\Phi}(t,\xi) \tag{20}$$

where $\boldsymbol{I}$ is the identity matrix with the appropriate dimension. According to Eqs.(17)-(19), it can be proved that the output and the state of the adjoint system Eq.(14) are expressed by

$$\boldsymbol{z}^{\mathrm{adj}}(\tau)=\boldsymbol{B}^{\mathrm{T}}(t_o-\tau)\boldsymbol{\Phi}^{\mathrm{T}}(t_o,t_o-\tau)\boldsymbol{C}^{\mathrm{T}}(t_o) \tag{21}$$

and

$$\boldsymbol{x}^{\mathrm{adj}}(\tau)=\boldsymbol{\Phi}^{\mathrm{T}}(t_o,t_o-\tau)\boldsymbol{C}^{\mathrm{T}}(t_o) \tag{22}$$

respectively, where $\tau\in[0, t_o]$. By combining Eq.(16) and Eq.(21), Eq.(15) can be obtained. Thus, the proof is completed.

Based on Theorem 1, this article proposes the main tool to compute the L_∞-induced norm of the LTV continuous system as follows.

Theorem 2 The closed-loop L_1 performance measure $J(\Sigma, t_f)$ of system Eq.(8) must satisfy the following equation:

$$J(\Sigma,t_f)=\sup_{t_o\in[0,t_f]}\int_0^{t_o}\left\|\boldsymbol{z}^{\mathrm{adj}}(t_o-\tau)\right\|_1\mathrm{d}\tau \tag{23}$$

where $\|\bullet\|_1$ is the column-sum norm of a matrix.

The proof of Theorem 2 is simple in the SISO case. Combining Eq.(11) with Eq.(15) results in the Eq.(23). It is also noted that when $\boldsymbol{w}(\tau)$ in Eq.(15) satisfies Eq.(24), the finite time infinite norm of the system output $\boldsymbol{z}(t)$ reaches the maximum.

$$\boldsymbol{w}(\tau)=\mathrm{sgn}(\boldsymbol{z}^{\mathrm{adj}}(t_o-\tau)) \tag{24}$$

where sgn(•) is a signum function that extracts the sign of a real number and $\tau\in[0, t_o]$.

It is important to point out that, a significant advantage over Eq.(13) lies in that the value of $\boldsymbol{z}^{\mathrm{adj}}(t_o-\tau)$ in Eq.(23) can be easily acquired through the auxiliary system Eq.(14). Moreover, Eq.(23) is suitable for the linear continuous system in such cases as finite horizon, infinite horizon, LTI, LTV, SISO, and MIMO. Also, in the finite horizon case, the system matrix $\boldsymbol{A}(t)$ in Eq.(8) may not be stable. Therefore, the proposed finite time L_1 approach is quite fit to analyze the missile terminal guidance system described by Eq.(7) since it turns unstable when the flight time t approaches the final intercepting time t_f.

For the LTI and SISO system, Eq.(23) can be simplified into

$$J(\Sigma,t_f)=\int_0^{t_f}\left\|z^{\mathrm{adj}}(t_f-\tau)\right\|\mathrm{d}\tau \tag{25}$$

Eq.(25) provides another method for computing the L_1 norm of a LTI system as well as for solving Eq.(12).

4. Missile Overload Requirement Analysis

The concept of the finite time L_1 approach proposed in Section 3 is of practical use to analyze the missile overload requirement in tackling the worst-case target maneuver since the guidance loop operates in a finite time interval. Consequently, it is necessary to analyze the relation between the guidance loop dynamics and the upper bound μ on the required missile-target maneuver ratio defined as

$$\mu\triangleq\frac{\sup\limits_{t\in[0,t_f]}|a_m(t)|}{\sup\limits_{t\in[0,t_f]}|a_t(t)|}=J(\Sigma,t_f) \tag{26}$$

In Ref.[4], let $\angle G(s)$ range from 0° to 180° in Eq.(6), then the maximum missile-target maneuver ratio μ is derived from

$$\mu=\frac{N}{N-2},\forall N>2 \tag{27}$$

However, to alleviate the negative effects of a high frequency noise, the phase of $G(s)$ in high frequencies should usually be required to drop rapidly under zero. In this respect, contrary to the existing literature, the proposed finite time L_1 approach can calculate the maximum missile-target maneuver ratio without any assumption on $G(s)$ and the detailed form of the worst target maneuver can also be obtained abiding by Eq.(24).

According to Eq.(11) and Theorem 2, in order to get the maximum missile-target maneuver ratio without loss of generality, only the normalized unit target acceleration needs to be taken into account.

The typical missile terminal guidance loop dynamics is described by Eq.(7) with the ensuing coefficients: t_f = 6 s, V_c=1 500 m/s, K = 0, T = 0.3 s, N = 4.

In Fig.3, assuming that the guidance dead zone is 300 m, the worst-case target maneuver form is shown over the finite time interval [0, 5.8]. The vertical axis denotes the normalized acceleration of the target. The missile maximum overload occurs at 5.8 s when the target performs the worst maneuver as shown in Fig.3.

Fig.4 shows the quantitative relation between the maximum missile-target maneuver ratio and the effective navigation ratio when T = 0.3 s. To avoid the larger missile overload requirement, N is expected to be in the interval [3.0, 4.5]. Compared to the results based on the conventional linear-quadratic performance index with terminal constraints, Fig.4 shows not only the optimal interval for the effective navigation ratio, but also the quantitative maximum missile-target maneuver ratio as well. Since the maximum target maneuver

ability is often known in advance, the missile maximum overload requirement can be obtained with the finite time L_1 approach.

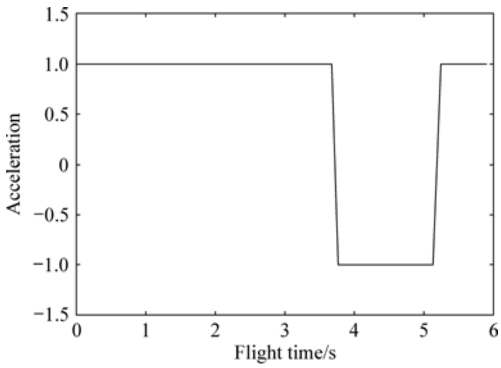

Fig.3 History of the normalized acceleration of the worst-case target maneuver.

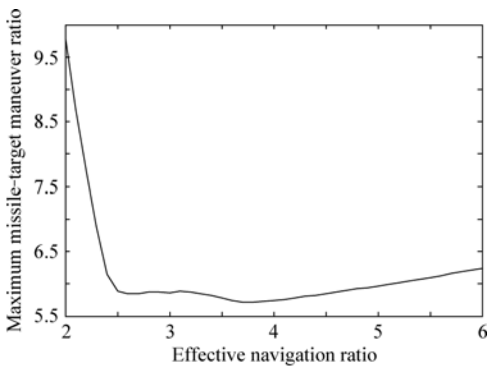

Fig.4 Maximum missile-target maneuver ratio with different effective navigation ratios.

Fig.5 shows that the guidance time constant T is the main contributor to the missile overload, and the maximum missile-target maneuver ratio is proportional to the guidance time constant. When T approaches zero, the maximum missile-target maneuver ratio equals 2, which accords with the result of Eq.(27) when $N = 4$. This indicates that reducing the response time of the missile/autopilot system not only delays the destabilizing moment of the terminal guidance system but also decreases the missile overload requirement.

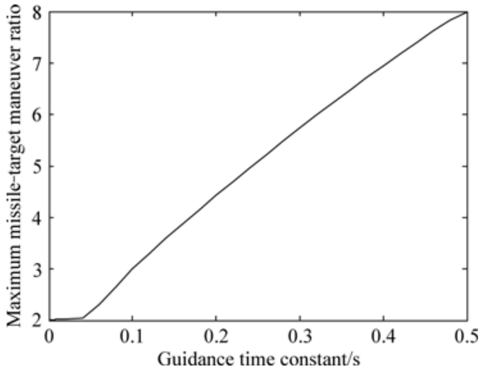

Fig.5 Maximum missile-target maneuver ratio with different guidance time constants.

Fig.6 clearly depicts that APNG is superior to the conventional proportional navigation guidance ($K = 0$) in that an appropriate modified coefficient to compensate the target maneuver can be chosen if the real-time estimated target acceleration is available, which leads to an enormous reduction of missile overload requirements. Fig.6 shows the optimal modified coefficient in APNG is 1.5.

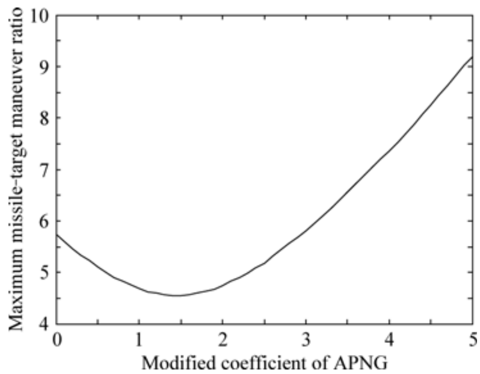

Fig.6 Maximum missile-target maneuver ratio with different modified coefficient of APNG.

Fig.7 compares three cases of the guidance loop dynamics. In Case 1, K, N and T equal 0, 4, 0.3 s, respectively. In Case 2, all parameters remain unchanged excepting the guidance time constant T which is 0.02 s. In Case 3, the better modified coefficients are chosen according to Figs.4-6, where K is 1.5. It should be stressed that the history of the target maneuver varies with flight time. When a proper guidance loop dynamics is designed, the maximum missile-target maneuver ratio can be decreased to 1.3.

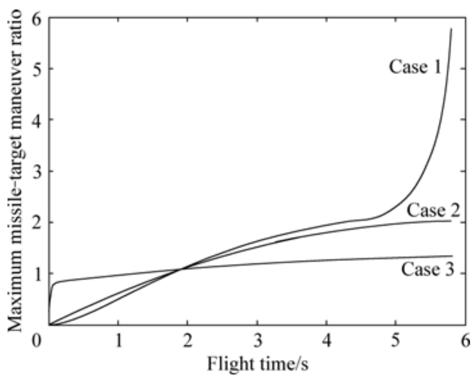

Fig.7 History of the maximum missile-target maneuver ratio.

From Figs.3-7, it is noted that the proper assessment of μ together with a proper design of the guidance loop dynamics may lead to a satisfactory nonsaturating guidance system.

5. Conclusions

A finite time L_1 performance measure is proposed for a LTV system, based on which the performance of

terminal guidance process with the various guidance loop dynamics is analyzed. The missile overload requirement for the terminal guidance is obtained and some design guidelines are given. It is emphasized that, within the proposed novel L_1 framework, the LTV continuous system is not required to be stable which is suitable for analysis of terminal guidance system.

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Biographies:

Ji Denggao Born in 1981, he received B.S., M.S., and Ph.D. degrees from Harbin Institute of Technology in 2003, 2005, and 2009, respectively. His main research interests include robust control, guidance and control of flight vehicles.
E-mail: jidg@163.com

He Fenghua Born in 1975, she received Ph.D. degree from Harbin Institute of Technology in 2004. Now she is an associate professor at Control and Simulation Center in the same university. Her main research interests include nonlinear control and hybrid control.
E-mail: hefenghua@hit.edu.cn

Yao Yu Born in 1963, he received Ph.D. degree from Harbin Institute of Technology in 1990. Now he is a professor at the Department of Control Science and Engineering in the same university. His main research interests include robust control, motion control, guidance and control of aircraft.
E-mail: yaoyu@hit.edu.cn